Acta Technica *62* (2017), No. 5A, 153–162

# Study on the gas emission law of cylinder-type coal cuttings based on time-varying diffusion coefficient

Cheng jiansheng[2,3,4], Yang huiming[3,4,5], Cheng bo[3,4]

**Abstract.** Gas emission law of coal cuttings is a key calculation influence factor to direct measurement of coal seam gas content, which is one of the basic parameters for gas disaster prevention and coalbed methane exploitation in underground coal mines. Due to gas emission calculation model error of coal cuttings in the direct method, the accurate determination of gas content remains a difficult problem. Most of the calculations adopt the gas emission model based on spherical coal cuttings. However, through the observation of the shapes of the coal cuttings after crushing and screening, we found that most of the coal cuttings shapes were cylindrical. In the paper, the time-varying characteristic of diffusion coefficient in the process of gas emission from coal cuttings was analyzed, and then the gas diffusion equation of cylinder coal was established based on the time-varying diffusion coefficient. At last, the reasonability of the analytical solution was verified by the method of experiment. The result showed that the analytical solution more accord with the result of the experiment test and make applicability more extensive compared with analytical solution of spherical coal cuttings gas emission under the steady diffusion coefficient. Besides its theoretical value, the analytical solution could be also served as a theory basis for accurate measurement of the gas content in the coalbed.

**Key words.** Mine safety, gas, diffusion, cylinder-type coal cuttings, analytical solutions.

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[1] Acknowledgment - This paper is supported by National Natural Science Foundation of China (Grant: 51604298, 51574280), and State Key Research Development Program of China (Grant: 2017YFC0804206).

[2] Workshop 1 - School of Energy Resources and Safety, Anhui University of Science and Technology, Huainan 232001, China;email:`jshcheng2005@163.com`

[3] Workshop 2 -State Key Laboratory of Gas Disaster Detecting, Preventing and Emergency Controlling, Chongqing 400037, China

[4] Workshop 3 -Chongqing Institute Co., Ltd. of China Coal Technology and Engineering Group, Chongqing 400037, China

[5] Corresponding author: Yang huiming ;email:`yhm3380@163.com`

## 1. Introduction

The coalbed methane (CBM) resources, also known as gas, is abundant in China and a new era of CBM exploitation will open up driven by favorable policies. Hence, it is of significance to research on the gas diffusion law in coal seams, which contributes to understand the CBM migration mechanism further, providing basis for its exploiting project. At the same time, it is more helpful in the testing method of gas content and prediction of coal and gas outburst in mines. Gas emission law of coal cuttings is a key calculation influence factor to direct measurement of coal seam gas content, which is one of the basic parameters for gas disaster prevention and coalbed methane exploitation in underground coal mines. Due to gas emission calculation model error of coal cuttings in the direct method, the accurate determination of gas content remains a difficult problem.

Generally, methane gas exists in internal holes and cracks of coal body in "adsorbed" and "free" forms[1−2]. However, once the original balanced state of "adsorption-desorption" breaks down, the gas in holes transmits into crack tunnels of coal gradually driven by the concentration gradient[3−5]. In researches of gas diffusion law of coal cuttings, most of scholars adopted combined method of theoretic analysis and experimental tests to investigate the diffusion behavior and effect of factors such as temperature[6], water, coal type[7], time[8,9], stress[10], vibration,etc. For example, Nandi[3]established diffusion model of spherical coal cuttings and acquires theoretic solution based on Fick's law of diffusion, which built foundation of its research for scholars. Afterwards, Charriere [4] analyzed the micro-mechanism of gas diffusion inside coal body and puts forward its pattern. Crank [5]took the influence of interfacial mass transfer into account and builds up corresponding mathematical diffusion model with its analytical solutions obtained. Rangel-German[7] analyzed the pore features of gas-fat coal and coking coal, then investigates adsorption and desorption characteristics of $CH_4/CO_2$mixed gas in coal body. Cui and Shi[8,9] pointed out that the functions of gas diffusion coefficient is related to time and the time-dependent rules of diffusion coefficient is gained by experiments.

Generally, the movement process of gas complies with Fick's diffusion law that diffusion speed is in direct proportion to concentration gradient. Whereas, coal is a kind of porous medium itself, which means that its porous construction changes with the decrease process of gas content[11]. Cui and Shi[8,9] verifies that diffusion coefficient is time variable during gas release of coal cuttings. Hence, it will reflect the real situation of its gas release while the diffusion coefficient is set as a time-dependent parameter in the classic model in research of gas release rule. Additionally, the present researches assume the coal cuttings as a sphere [3,8,9]. While, the author found that the coal cuttings are round or planer shaped and mostly cylinder and cuboid when it's observed by ThunisA688 scanner after coal's crushing, which is shown in Figure 1. Therefore, it will definitely lead to a discrepancy between theoretic analysis and experimental results without considering the influence of time-dependent diffusion coefficient and the shape of coal cuttings. In the meantime, the shape of planer and cuboid is close to cylinder, thus the gas release rule of cylinder shaped coal cuttings is closer to real situation. Based on this finding, the paper analyses the time dependent features during gas release firstly, and establishes gas diffusion equation. Secondly, the analytical solution is gained by method of mathematical physics, and then the solution is further verified by experiment result. The research would benefit the insight about diffusion behavior, and the quick determination of coalbed gas content and predictive index of coal and gas outburst.

Fig. 1. Observation graphic of cinder

## 2. Theoretic analysis of gas release features based on time-dependent diffusion

The spread of gas in coal cuttings is a result of random motion of gas molecule essentially, in which it transfers from high concentration zone to the lower driven by concentration gradient. The essence of diffusion coefficient reflects the migration restriction which gas molecule moves in coal cuttings. In the early period of release, there exists a discontinuous potential difference along the travel direction of gas molecule. Afterwards, the concentration distribution reaches a continuous balanced state with the movement and collision of gas molecule.

At first, gas in coal cuttings is assumes as ideal one and the collision of gas molecule in coal's faces is elastic specular reflection. Thus, the mean velocity is as followed.

$$\bar{v} = \sqrt{8RT/\pi M} \tag{1}$$

Where, $R$ represents gas phase constant; $T$ represents thermodynamic temperature; $M$ represents gas molecule weight.

Suppose that a square shaped diffusion element exists in coal cuttings, and the volume that gas molecule pass through the section of $A$ area per unit time is $\bar{v}A$. Moreover, make an assumption that the number density of gas molecule is $n_w$ and its movement probability is equal in 6 directions, then the molecule flow through this section is $J = \frac{1}{6}\bar{v}An_w$ per unit time. The essence of Fick's diffusion rule is to describe the macro statistic features that shows during gas movement, that is $J = D\frac{dn_w}{dl}A = \frac{1}{6}\bar{v}An_w$. As a result, during gas release, the gas molecule density and gradient value will drop along diffusion direction. However, the drop rate of the former is bigger than the latter and consequently the macro statistic features shows that gas diffusion coefficient decreases constantly. While in the later period, the gas molecule density has dropped to the minimum value and its variation rate is close to a constant one, thus the corresponding gradient value along diffusion direction tends to be a limit one. Therefore, the gas diffusion coefficient tends to reach a constant value with the extension of time.

## 3. Gas release model of cylinder shaped coal cuttings

The coal cuttings are supposed to be a homogeneous cylinder with a radius of $R_0$ and a height of $2h_0$. The gas concentration in coal cuttings is $C_0$ in the state of balanced adsorption. Then, the concentration on the faces of coal cuttings is $C_1$ while the atmosphere drops to 1 bar suddenly. In that case, the gas diffuses toward the ring and axial direction of cylinder from the inside, in which the diffusion coefficient $Dt$ is related to time. As shown in Figure 2.

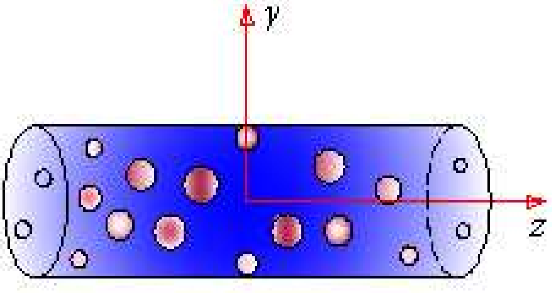

Fig. 2. Sketch map of cinder gas emission of cylinder coal

The mathematical model of gas release is :

$$\begin{cases} D_t\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right)=\frac{\partial C}{\partial t} \\ C\left(r,z,0\right)=C_0; C\left(\pm R_0,\pm h_0,t\right)=C_1 \\ \frac{\partial C}{\partial r}\Big|_{r=0}=0;\ \frac{\partial C}{\partial z}\Big|_{z=0}=0 \end{cases} \tag{2}$$

To simplify the solution process, $C_N$is defined as $C_N$=$C$-$C_1$which is then taken into equation (2) and the same integral transform processing is made on time-dependent diffusion coefficient $D_t$in equation (2). Suppose $f\left(t\right)=\int_0^t D_t dt$ then $D_t=\frac{df(t)}{dt}$ and the original equation turns into:

$$\begin{cases} \left(\frac{\partial^2 C_N}{\partial r^2}+\frac{1}{r}\frac{\partial C_N}{\partial r}+\frac{\partial^2 C_N}{\partial z^2}\right)=\frac{\partial C_N}{\partial f} \\ C_N\left(r,z,0\right)=C_0-C_1 \\ \frac{\partial C_N}{\partial r}\Big|_{r=0}=0;\ \frac{\partial C_N}{\partial z}\Big|_{z=0}=0 \end{cases} \tag{3}$$

Assume that the solution of $C_N$in equation (3) meets the form: $C_N$=$S(r).H(z).L(f)$

and $S(r)H(z)$ $L(f)$ is merely the function about $r$ $zf$. In addition, take $C_N{=}S(r).H(z).L(f)$ into equation(3) and obtain that:

$$\frac{1}{S}\frac{\partial^2 S}{\partial r^2}+\frac{1}{Sr}\frac{\partial S}{\partial r}+\frac{1}{H}\frac{\partial^2 H}{\partial z^2}=\frac{1}{L}\frac{\partial L}{\partial f} \tag{4}$$

If equation (4) holds up, the both sides equal to a constant. If it is set as -$\varepsilon^2$and the solution about f can be acquired:

$$L\left(f\right)=A_2e^{-\varepsilon^2 f} \tag{5}$$

Where, $A_2$is integration constant.

Suppose that -$\mu^2$brings equation (6) into existence:

$$\frac{1}{H}\frac{\partial^2 H}{\partial z^2}=-\varepsilon^2-\frac{1}{Sr}\frac{\partial S}{\partial r}-\frac{1}{S}\frac{\partial^2 S}{\partial r^2}=-\mu^2 \tag{6}$$

Then:

$$H=B_1\cos\left(\mu z\right)+B_2\sin\left(\mu z\right) \tag{7}$$

Where, $B_1$??$B_2$is are integration constants.

Bring the solution of $H(z)L(f)$ into $C_{N,}$then:

$$C_N=e^{-\varepsilon^2 f}\left[B_3\cos\left(\mu z\right)+B_4\sin\left(\mu z\right)\right].S\left(r\right) \tag{8}$$

Where $B_3$??$B_4$is the product of $A2$ and $B1$??$B2$ separately.

Take the boundary conditions of equation (3) into (8), it follows that $B_4{=}0$. So, $C_N$is gained:

$$C_N=e^{-\varepsilon^2 f}.B_3\cos\left(\mu z\right).S\left(r\right) \tag{9}$$

Because the value of $\mu$ can be innumerable, and it can be acquired according to the theory of solution's linear superposition:

$$C_N=\sum_{n=1}^{\infty}e^{-\varepsilon^2 f}.B_{3n}\cos\left(\mu_n z\right).S\left(r\right) \tag{10}$$

The first item in the right side of equation (6) can be written as:

$$\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}+\left(\varepsilon^2-\mu_n^2\right)S=0 \tag{11}$$

Equation (11) is a typical zero-order Bessel equation and its general solution is:

$$S\left(r\right)=N_1J_0\left(\sqrt{\varepsilon^2-\mu_n^2}r\right)+N_2Y_0\left(\sqrt{\varepsilon^2-\mu_n^2}r\right) \tag{12}$$

Where, $N_1N_2$are the integral constants; $J_0\left(\sqrt{\varepsilon^2-\mu_n^2}r\right)Y_0\left(\sqrt{\varepsilon^2-\mu_n^2}r\right)$ are the first and second kind of zero-order Bessel equation.

While $r{=}0$, the value of the second kind zero-order Bessel equation shall tend to be infinite in equation (12). This will definitely lead to a disagreement with the actual initial conditions. So, the integral constant $N_2$ should be set as 0. Then:

$$C_N = \sum_{n=1}^{\infty} e^{-\varepsilon^2 f}.B_{3n} \cos\left(\mu_n z\right).N_1 J_0\left(\sqrt{\varepsilon^2 - \mu_n^2} r\right) \tag{13}$$

Likewise, it can be obtained according to the theory of solution's linear superposition:

$$C_N = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} B_{3n}.N_{1m} e^{-\varepsilon^2 f}.\cos\left(\mu_n z\right).J_0\left(\sqrt{\varepsilon^2 - \mu_m^2} r\right) \tag{14}$$

Take the initial conditions in equation (3) into (14). Then,

$$C_0 - C_1 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} B_{3n}.N_{1m}.\cos\left(\mu_n z\right).J_0\left(\sqrt{\varepsilon^2 - \mu_m^2} r\right) \tag{15}$$

Make integral treatment on equation (15):

$$\begin{gathered} B_{3n}.N_{1m} = \left(C_0 - C_1\right).\tfrac{2\sin(\mu_n h_0)}{[\mu_n h_0 + \sin(\mu_n h_0).\cos(\mu_n h_0)]}. \\ \tfrac{2J_1\left(\sqrt{\varepsilon - \mu_m^2} R_0\right)}{\sqrt{\varepsilon - \mu_m^2} R.\left[J_0^2\left(\sqrt{\varepsilon - \mu_m^2} R_0\right) + J_1^2\left(\sqrt{\varepsilon - \mu_m^2} R_0\right)\right]} \end{gathered} \tag{16}$$

Bring equation (16) into (14) and it is the gas concentration distribution of cylinder shaped coal cuttings during release process. Take the radius and height of coal cuttings into the analytical solutions of concentration distribution and the gas release amount according to different time can be obtained.

## 4. Experimental verification

As shown in Figure 1, the shape of coal cuttings is multiple after the crushing and screening of coal sample and it is mostly cylinder or cuboid instead of spherical. Literature[12] confirms that the statistical distribution of particle size of crushed coal is a skewed one. When the weight of coal sample is over 15kg, the statistical particle size tends to be stable. The coal sample used in this article is taken from 27131 working face in Mazhuang cokes mine in Henan Province, whose ash content is 17.52%, volatile component is 8.74%, true density is 1.65g/cm$^3$, apparent density is 1.57g/cm$^3$, utmost adsorbed amount is 43.6851cm$^3$/g and the adsorption constant $b$ is1.6471 MPa$^{-1}$. After the crushing and screening operation, the coal cuttings with the grain size of 0.2˜0.25mm is adopted and its total weight is 20.003kg. Afterwards, apply the statistical plot function in MATLAB software to conduct Weibull distribution inspection and solve the related parameters about it by square estimation and empirical method. As a result, the stable shape of coal cuttings is a cylinder with a diameter of 0.18mm and a length of 0.24mm. In this experiment, the balanced adsorption pressure is 0.83MPa and the experimental temperature is 30??. Moreover, the testing method is identical with literature [5], so there is no need to make a statement here. Additionally, the computation method of diffusion coefficient is similar to that of spherical coal's gas diffusion coefficient. However, the difference is that the diffusion coefficient of cylinder shaped coal cuttings is supposed to be a constant at first. Then, it can be figured out by the obtained concentration distribution and ultimate gas release amount. It will not be introduced owning to a space limit. The experiment result is displayed in Figure 3.

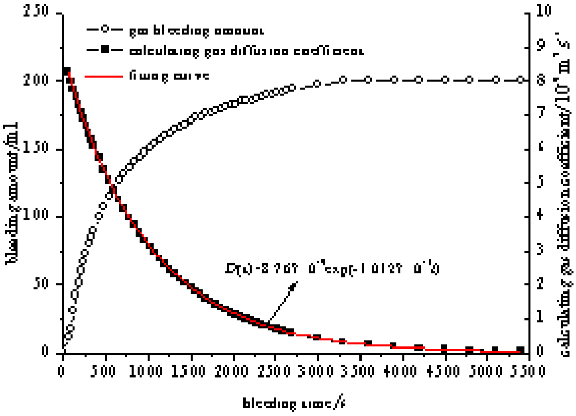

Fig. 3. Emission rate and diffusion coefficient in different time

From figure 3, it can be seen that the gas diffusion coefficient of coal sample takes on a decline feature with the extension of release time, which conforms to the research findings of predecessors. And it is found that both of them comply to a function by fitting diffusion coefficient at different time:

$$D\left(t\right)=D_0\exp\left(\varphi t\right) \tag{17}$$

Where, $D_0$is the initial diffusion coefficient and its value is $8.76\times10^{-9}\mathrm{cm}^2.\mathrm{s}^{-1}$;$\varphi$ is the attenuation coefficient and its value is $-1.012\times10^{-3}.\mathrm{s}^{-1}$.

Equation (17) is consistent with the dynamic diffusion coefficient proposed by literature [5]. Furthermore, inspect and compare the spherical coal's gas release rules according to the time-dependent diffusion coefficient represented by equation (17). The compared spherical coal cuttings is 0.2mm??0.25mm in diameter and the results is shown in Figure 4.

From Figure 4, the change characteristic of computed diffusion coefficient of spherical coal cuttings which are 0.2mm??0.25mm in diameter is same with that of the cylinder. Whereas, the concrete number is different where the computed diffusion coefficient of 2.0 diameter spherical coal grain is the biggest and that of cylinder is the second to it. Moreover, the computed release amount by spherical coal cuttings has a big deviation from the actual one and that of the cylinder model based on time diffusion coefficient proposed by this article is more in line with the actual. It indicates that the pick of coal's shape influences the computed gas diffusion coefficient and the physical property parameter affects more on its grain size and shape after its crushing and screening. Thus, its shape should get strictly divided while studying the gas release rule of coal cuttings. In that case, the theory results

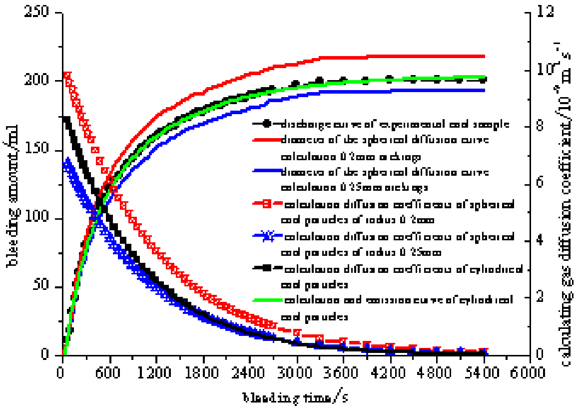

Fig. 4. Comparison curves of cinder diffusion with cylinder and sphere coal

can be closer to real situation and it is beneficial for the improvement of gas diffusion theory further. What's more, it is of great significance for the research of quick gas content determination method in coal seamsand predictive index of coal and gas outburst.

## 5. Conclusions

(1) It is found that most of coal cuttings are spherical, planar or cuboid from images of the crushed cuttings. Owning to that the shape of a plate or cuboid is close to cylinder, gas release law of cylinder coal cuttings shall more accord with real situation.

(2)The mathematical model of cylinder shaped coal cuttings based on time-dependent diffusion coefficient is established, and its analytical solution is obtained by method of mathematical physics. This can be used to fit experimental statics and provide theory basis of gas determination method under the shaft.

(3) The obtained analytical solution is verified by experiments in laboratory and the results show that the gas diffusion model of cylinder shaped coal cuttings based on time-dependent diffusion coefficient is closer to real situation compared to that of the spherical one. Therefore, in research of gas release rules of coal cuttings, a rational pickup of mathematical model should be made according to the shape of crushed coal to help uncover mechanism of gas release further.

(4) As the gas emission behaviour of coal is affected by kinds of factors, the research based on the new shape assumption in the paper is just a beginning of series of research. Based on new shape assumption, the gas diffusion research under the combination effect of factors such as temperature, water and coal type etc. should be the future study direction to reveal diffusion mechanism and to describe precisely the gas diffusion behaviour.

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Received November 16, 2017